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COMMENT

Comment on 'On the complete integrability of the Hirota–Satsuma system'

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Abstract. We show that the aforementioned paper by Chowdhury and Mukherjee is false.

In their paper, Chowdhury and Mukherjee (1984) claim that the Hirota–Satsuma (1981) equation fails the Painlevé test. They use the method proposed by Weiss *et al.* (1983), but they do not compute the resonances. After their equation (10) they claim that a set of equations is not mutually consistent and they conclude that the system fails the Painlevé test. In the last part of their paper they use a similarity reduction that leads to two coupled third-order ordinary differential equations, and they claim that it is 'a matter of routine' to see that this set does not belong to the Painlevé class classified by Ince (1956).

This last claim is difficult to understand since Ince (and Painlevé) only classified one-component second-order equations. Even a one-component *third*-order equation, let alone coupled third-order equations for two components, i.e. a sixth-order system, will not be present in Ince's classification whether they have the Painlevé property or not.

To go back to the first part of their paper we note that it contains calculational errors.

In equation (5) α is a new parameter, quite distinct from $\alpha = -2$ in equation (2). This new α should satisfy

$$b\alpha^2 + 24a = 0$$

a relation that never appears in the paper on which we are commenting. Consequent upon errors in equations (3) and (4) the equation (6) and indeed almost all subsequent equations are incorrect. One can easily convince oneself that besides the $\phi_x^3 u_2$ term the only terms that can appear must be homogeneous to ϕ^3/x^5, with t counting as x^3. This means that the only possible terms are

$$\phi_x^2\phi_t, \qquad \phi_x^2\phi_{xxx} \qquad \text{and} \qquad \phi_{xx}^2\phi_x.$$

We did not check the coefficients of these terms, but certainly the terms

$$6\phi_{xx}^2, \qquad 6\phi_x\phi_{xxx} \qquad \text{and} \qquad -18\phi_x^3\phi_{xx}$$

cannot be present.

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Similar considerations apply to equation (8) which should take the form

$$(48au_2-4\alpha bv_2)\phi_x^3+\lambda\phi_x^2\phi_t+\mu\phi_x^2\phi_{xxx}+\nu\phi_{xx}^2\phi_x=0,$$

there λ, μ and ν are numbers that depend on a and b (and α, but α^2 is fixed by a and b). We did not bother to compute them because they are irrelevant for our purpose and the calculations are very ungainly. However, it is easy to convince oneself that no other terms can arise. Thus both u_2 and v_2 are determined contrary to the assertion of the authors.

We can make several more remarks.

(i) One leading behaviour was missed, namely

$$u\sim-2\phi_x^2/\phi^2, \qquad v\sim v_1/\phi, \qquad v_1 \text{ free.}$$

(ii) In the leading behaviour that was considered

$$u\sim-4\phi_x^2/\phi^2, \qquad v=\alpha\phi_x^2/\phi^2,$$

two resonances were ignored, namely 6 and 8.

Although this could only put further constraints on the Painlevé analysis, it turns out that, contrary to what is claimed, the Hirota–Satsuma equation (for $a=\frac{1}{2}$ only) does pass the Painlevé test. Indeed, for both leading behaviours, and for all resonances, the resonance condition is satisfied, as we have shown in an earlier publication (Ramani *et al* 1983). We used a simpler algorithm than the Weiss one, which is due to Kruskal (1982) (see also Jimbo *et al* 1982) that made the check possible up to order 8. Due to the awkwardness of the algorithm it could not be seen that the equations are in fact compatible. We did not perform the calculations using Weiss's algorithm beyond order two. However, errors had already been introduced at this stage. This algorithm, however, if used properly would necessarily give a positive answer since it is completely equivalent to the much more efficient Kruskal algorithm, and the answer given by the latter is positive. We repeat that, as we have published earlier (1983) the Hirota–Satsuma equation (for $a=\frac{1}{2}$) passes the Painlevé test and is presumably integrable.

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